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13. ABSTRACT (Maximum 200 words) This report summarises accomplishments from Phase 3, the objectives of which are: (1) the refinement and evaluation of our constitutive models and (2) the implementation of our constitutive models using the finite element method to the classical flat punch problem. To the best of our knowledge, our particular micromechanical approach has delivered the first breed of enriched continuum models for granular media, with a resolution high enough to capture internal microstructures that are only a few particles wide, e.g. shear bands. In addition, these models have a number of distinct advantages over other models of granular media (e.g. the ability to capture loss of contacts, slip and non slip modes of inter-particle contact, evolution of contact and force anisotropy, interparticle rolling resistance etc.) Model predictions against two benchmark laboratory experiments (viz. bi-axial compression and simple shear of assemblies of uniformly sized circular rods) were in good agreement with experimental data, particularly, on the phenomenon of shear banding. In fulfilment of the second objective for Phase 3, we also examined the frictional contact of a rigid flat punch and found our model predictions to be consistent with experimental findings. Research efforts this year will focus on the finite element implementation of our micromechanical constitutive models to a broader range of engineering scale problems, including sand-tire interaction.				
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Enclosure 1

Final Project Report
Period of Performance December 1, 2002 - November 30, 2004

Toward generalized continuum models of granular soil and soil-tire interaction
Proposal Number 42855-EV

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The University of Melbourne

1. Objectives

The project objectives are to:

- Develop micromechanical constitutive models of dry granular materials. These models have the potential to offer a computationally efficient alternative to DEM simulations, and fill an important niche in applications beyond the reach of current DEM simulations (*e.g.* soil-structure/machine interaction systems).
- Experimentally validate these constitutive models.
- Integrate these constitutive models into our contact mechanics model of the soil-tire interaction system.

2. Research Background

The great majority of existing models of granular media are developed using classical continuum theory [1,2]. These models are readily accessible and enjoy widespread use. However, their Achilles heel is that they possess no length scale with which to accommodate microstructural properties governing bulk behaviour of granular media [3]. Thus, the reliability of these models remains questionable. An alternate approach, which is gaining popularity, is the discrete element method (DEM) [4,5]. DEM deals explicitly with the microstructure and, as such, has delivered reliable predictions on granular behaviour. However, the number of particles in current DEM simulations is limited to a few million particles – *roughly the number of particles in a handful of sand!* Thus, many engineering scale processes are beyond the reach of DEM, while prototype scale problems require the use of oversized particles, which leads to scaling errors [4,5]. Recently, effort has been directed toward formulating a third class of models with the combined strengths of classical continua and DEM models [3,6-8]. It is this hybrid that we have been developing using Micromechanics theory [7,8,13-17,20]. The basic idea in micromechanics is to relate the microstructural/*discrete* properties (*e.g.* contact forces, contact moments, displacements and rotations), to macrostructural/*continuum* properties (*e.g.* stresses, couple stresses, strains and curvature) using a “homogenisation” or averaging procedure.

3. SEEING THE DISCRETE IN A CONTINUUM: Significance of the research and its innovative aspects

The Holy Grail in the constitutive modelling of granular media is to understand the physics that governs material behaviour – from the microscale to the continuum scale. Our group has recently pioneered the development of a novel micromechanical approach on multiscale constitutive modelling of granular media, which weaves together recent advances in modern continuum theory (*i.e.* Thermomechanics [11,12], Micropolar Theory [7,10], Homogenisation Theory [9] and Micromechanics [3,6,9]), classical continuum theory, large-scale discrete element methods (DEM) [4,5], and experimental micromechanics [3]. The foundation of this new methodology was

published in *Powder Technology* in 2002 [7]. In 2003, we developed the first breed of high-resolution continuum models of dry granular media with predictive capability on both local behaviour (i.e. fine micro-scale structures such as shear bands) and global behaviour. Our analysis was confined to dry assemblies of uniformly sized circular particles. We found excellent agreement between our model predictions and experimental observations of uniformly sized assemblies of Schneebeli rods and photoelastic disks [16,17,20,8]. In addition, these continuum models have successfully captured *emergent behaviour* – specifically, contact and contact force anisotropies – without need for any “fabric and force evolution laws” to be introduced into these models. This entirely new level of predictive capability is unmatched by any other continuum model developed to date. Six papers have been accepted for publication in top scientific journals from a wide cross section of disciplines engaged in granular media research: *Powder Technology*, *Acta Mechanica*, *Granular Matter*, *International Journal for Analytical and Numerical Methods in Geomechanics*, *BIT Numerik Mathematik*, *International Journal of Solids and Structures*. In addition, we have four more papers currently in review: one in *Geotechnique*, one in the *Journal of Engineering Mathematics* and two invited papers in the *Proceedings of the International Congress for Theoretical and Applied Mechanics 2004*.

Reviews of our accepted papers indicate that this theoretical innovation is “*one of the very few which is making a serious attempt to bridge the gap between micromechanical studies of granular media and the application of continuum mechanics on the macro-scale*” and is said to parallel recent developments on porous media, thereby proving the general applicability of these techniques in the Science of Complex Media and composites. Further recognition of the importance of this development, internationally, is evidenced by an invitation to give a plenary talk on multiscale modelling at the International Conference on Scientific Computation and Differential Equations (SciCADE’03, Norway, June 30 - July 4, 2003).

Advantages of our continuum micromechanical models and its novel aspects are:

- A correct level of resolution for capturing *emergent* fine-scale microstructures whose characteristic length scales are only a few particles wide, *e.g.* shear bands and force chains. In standard micromechanics theory, such microstructures are “smeared out” in the modelling process, as the averaging domain (representative volume element or ‘RVE’) in traditional homogenisation methods consists of many thousands of particles [3,6,7,9]
- A clear link between macro and micro behaviour is established. These new models have predictive capability of bulk behaviour, based on physical properties of the particles and their interactions (*e.g.* particle stiffness coefficients, coefficients of inter-particle rolling and sliding friction) [13-17]. This is in contrast with the great majority of continuum phenomenological models whose material parameters, which are obtained from “curve-fitting” analysis, have no direct physical meaning [18].
- Since these models have the same input material parameters as DEM models, direct comparisons could be made not just with experiments, but also with DEM simulations. This is vital, given that DEM remains the surrogate for experimentation on internal microstructural mechanisms, while non-invasive experimental methods remain severely limited [3].
- The models developed within this framework are guaranteed to be thermodynamically viable. This eliminates the need for checking *a posteriori* that the models are objective and obey the laws of thermodynamics. Furthermore, because no additional constraints need to be introduced, the generalised constitutive relations describe the broadest possible range of thermodynamically admissible models [13].
- Experiments have shown that local fluctuations in both particle displacements and rotations bear a significant influence on the bulk behaviour of granular media – *even at low to moderate strains* [13,19]. By appealing to the notion of “internal state variables”, our thermomechanics based approach offers a rigorous method for the treatment of such local

scale kinematic fluctuations, as well as other state variables, in a thermodynamically consistent manner [11-13, 20].

- Conventional micromechanical models require, *a priori*, the evolution of anisotropies for input. The drawback of this approach is that it constrains the model to a predefined mode of deformation – effectively eliminating the emergence of novel anisotropies. In contrast, the proposed approach obviates the need for assumed evolution laws to be introduced into the model. Instead, more sophisticated contact laws are adopted, which enable the model to correctly predict the evolution of anisotropies. Thus, in our approach, the evolution of anisotropies becomes an output of, instead of an input to, the model [8,15,16].
- Enriched continuum models are more computationally efficient than DEM models. DEM simulations are limited by computer hardware. Currently, simulations are limited to a few million particles (roughly the equivalent of a handful of sand). Therefore, many engineering processes (e.g. soil-structure interaction problems) are beyond the reach of DEM analysis, while use of oversized particles to solve prototype scale problems leads to scaling errors in the solution [4,5]. In fact, Peters has suggested that: “*Unless computing power reaches a point where there is a one-to-one correspondence between simulated particle size and actual particle size, some understanding of the DEM medium as a continuum will be needed.*” [4].

4. Approach

Over the past few years, we have made important advances in developing the first breed of enriched continuum models for high resolution analysis of particulate media. The approach we have developed fuses together techniques and principles from both modern and classical mechanics: *viz.* Thermomechanics, Micromechanics, Micropolar or Cosserat theory, Homogenisation Theory.

4.1 High resolution homogenisation technique

Key micro-structures of only a few particles are “smeared out” in traditional micromechanical homogenisation methods [3,6,7,9]. A novel feature of our recently developed approach is a high resolution homogenisation method, devised on the scale of a particle and its immediate void space, as shown in Figure 2 [7,8]. In essence, continuum properties are derived from a statistical average of the discrete interactions occurring between a particle and its first ring of contacting neighbours. In this homogenisation scheme, the fabric is represented geometrically by a number of tessellation methods (*e.g.* Voronoi and Dirichlet) [3]. Fabric properties (such as void ratio, disorder, contact anisotropy, and number of contacts) of a particle assembly can be obtained from the statistics of cells in the tessellation representation; similarly, the evolution of fabric can be inferred from the evolution of the tessellated structure.

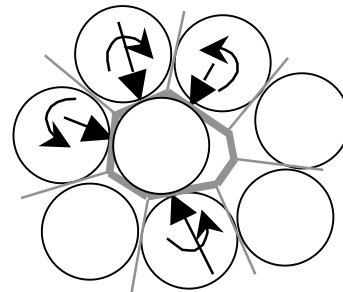


Figure 2: Homogenisation scheme averages interactions with contacting neighbours over the area of the Voronoi cell.

4.2 Modelling techniques with fabric evolution as an emergent property

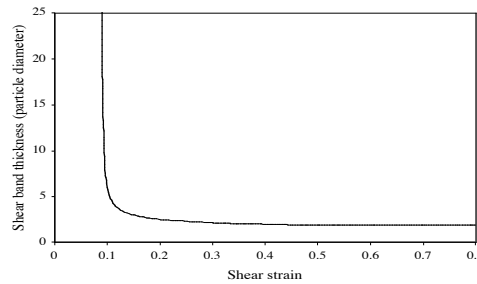
Conventional micromechanical models depend on *a priori* defined fabric and fabric evolution laws (see pgs. 230-233 of [3]). Our research has shown that it is possible to develop models which do not require fabric evolution laws, provided the most essential physics governing interparticle interactions are correctly captured in the modeling process [8,15,16]. We have successfully developed models, which, like DEM, can correctly predict the evolution of deformation-induced contact and contact force anisotropies, without need for assumed anisotropies to be imbedded into the model. This was achieved by linking the interparticle contact laws (defining the different modes

of contact, both sliding and non-sliding) as well as the condition for loss of contact, with the local strain. A mean-field approximation to the relative motion at a contact, subject to sliding and rolling friction, was adopted to relate the local strain to the contact laws and the condition for loss of contact. *The achievement of a model of this capability is a significant step forward, since the introduction of deformation-induced anisotropies into the modeling will constrain the model to a predefined mode of deformation, effectively eliminating the emergence of novel anisotropies. What happens then, when real conditions fail to conform to these assumed modes of deformation?*

4.3 How faithful are these enriched continuum models to discrete granular assemblies?

The advances outlined above were derived from studies of two-dimensional, dry assemblies of uniformly-sized, circular particles. Our analysis focused on small strain behaviour of assemblies under quasi-static loading conditions. To assess our models' predictive capabilities, we examined

Figure 3: Predicted shear band evolution: final thickness consistent with observed thickness values of 2-4 particles in real assemblies of uniformly sized circular rods.



the properties of *real* mono-disperse assemblies under these conditions in various benchmark experiments, including: (a) bi-axial compression test, (b) simple shear test, (c) contact with a rigid flat punch. In every case, we found good

agreement between model predictions and experimental data. Figures 3-5 show our model predictions for shear band formation in bi-axial compression tests. There is excellent agreement between these results and experimental observations on assemblies of photoelastic discs (Figure 6), and Schneebeli rods [3,19]. The anisotropies resulting from an *initially isotropic* system are shown in Figures 4 and 5. Although our model formulation can accommodate a pre-defined evolution of contact and contact force anisotropy, we found that our strain-dependent laws for contact and loss of contact are sufficient to produce experimentally observed evolution of anisotropies in initially isotropic systems. To demonstrate, we show in Figure 6 the two stages of shear banding in a bi-axial compression test. First, the granular assembly adopts configuration (a) to provide the maximum resistance to the applied vertical compression: here, near vertical particle columns are formed and the maximum normal contact forces are aligned with these columns (cf. Figures 4(a),5(a)). Under continuing vertical compression, the assembly then adopts configuration (b): here, the particle columns “buckle” to form a so-called shear band that is characterised by a pattern of interlacing particle rows (almost perpendicular to the band axis) and voids. In configuration (b), the contact and normal contact force anisotropies around a particle inside the band are inclined to the vertical direction (cf. Figures 4(b),5(b)).

Figure 4: Polar plots of normal and tangential component of contact force around a particle: (a) before shear banding, (b) inside a shear band. Thin (thick) line represents the normal (tangential) contact force. *Direction of maximum normal contact force indicates direction of force chains.*

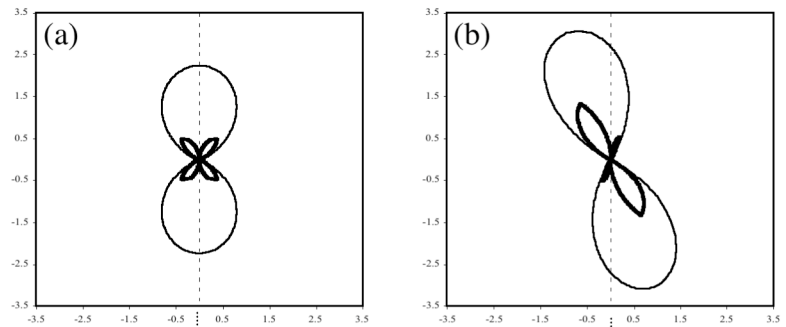


Figure 5: Distribution of contacts around a particle: (a) before shear banding, (b) inside a shear band.

no contact;
 sliding +ve
 non-sliding;
 sliding -ve

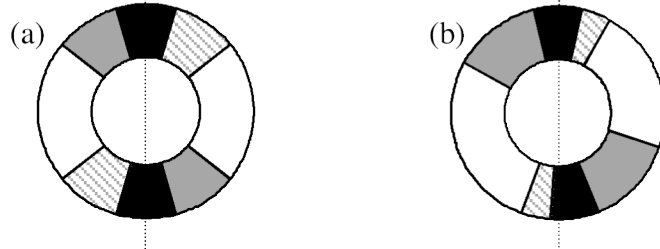
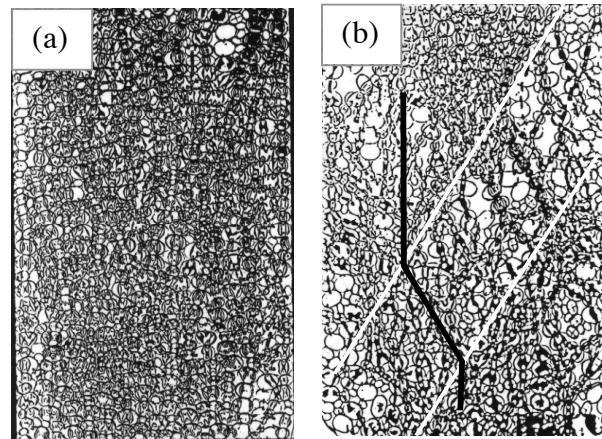


Figure 6: Bi-axial compression of randomly packed photoelastic discs: (a) before shear banding -the particle columns and direction of maximum stress transmission (force chains) are aligned with the direction of applied vertical compression; (b) a partially formed band emanating from top right corner. White lines were drawn in to indicate the boundaries of the shear band once fully developed; black buckled line drawn in to indicate the form of a buckled particle column. *Inside the band, particle columns and force chains are inclined to the vertical.*



5. Accomplishments

In 2003, we carried out Phase 3 of our research program. The objectives in Phase 3 are the refinement and evaluation of our micromechanical constitutive models and the implementation of these models to the analysis of certain benchmark problems, both laboratory scale (e.g. bi-axial compression, simple shear) and engineering scale problems (e.g. flat punch analysis). This was achieved through a series of projects as follows:

Project 1. Micromechanical modelling of two-dimensional dry granular materials

Recent research conducted by our Granular Mechanics group has concentrated on developing techniques for relating microstructural properties (particle-scale) to macroscopic (or bulk) properties. This procedure is called homogenisation. Using the homogenisation scheme developed by Tordesillas and Walsh [7] for two-dimensional (2D) assemblies of uniformly sized circular particles, we have developed constitutive laws which account for sliding and non-sliding contacts, rolling resistance at contacts, and loss of contacts. The analysis was performed assuming the granular material is a micropolar or Cosserat continuum, thereby allowing each material point to have both translational and rotational degrees of freedom. Furthermore, the use of strain dependent contact laws has resulted in a constitutive law that is able to predict the evolution of microstructural properties such as the degree and direction of contact anisotropy and contact force anisotropy.

Key deliverables: publications [1-13], conference presentations [14-21]

Project 2. One-dimensional shear band analysis using generalised micropolar constitutive laws

In this project, the problem of shear band formation in a biaxial test was examined to test the capabilities of the constitutive models developed in Project 1. The problem of shear band formation was chosen due to the relative wealth of experimental and discrete simulation results in the literature. However, before this could be done, an analytical model of the biaxial test and shear band formation had to be formulated. The one-dimensional (1D) shear band analysis, first introduced by Mühlhaus and Vardoulakis in 1987, was adopted and modified. Therefore, there were *two* models involved in this analysis: the 1D model for the shear band and the micromechanical constitutive model for the granular material. In assessing the predictive capabilities, one must therefore distinguish between limitations that are due to the 1D shear band model as opposed to those limitations that are due to the micromechanical constitutive model. Hence, a generalised micropolar constitutive model, which had as a subset our micromechanical model along with a wide range of other models, was examined within the 1D shear band analysis. This generalised constitutive model also paved the way for the testing of any of our future constitutive models.

Key deliverables: publications [1,3,4,6,8,9], conference presentations [14-17]

Project 3 Software development: Visualisation for discrete element simulations.

The discrete element method (DEM) simulation is a technique that is often used as pseudo experiments in granular research. In this technique, the equations of motion for up to several million particles are determined individually within a time integration algorithm. This method allows all information about every particle to be known throughout the life of the simulation. Hence, data handling (storage, manipulation, visualisation and statistics) becomes a key issue. Project 3 involved developing an advanced visualisation and statistical software to aid the *viewing* of DEM output. Acting as clients for a team of 15 4th year computer science students from The University of Melbourne, we provided an extensive list of requirements and continuous feedback and technical advice.

Key deliverables: visualisation software PVS, publication [11], conference presentation[20]

Project 4. Micromechanical modelling of three-dimensional wet foams

Although the homogenisation scheme was originally developed for granular media, it can also be applied to other complex systems. To demonstrate the flexibility of our technique, we are currently using the homogenisation procedure to develop deformation models for three-dimensional (3D) wet foams, and in the process expose the close links between seemingly diverse materials. 3D wet foams are defined by foams with a liquid content near 25%, such that the foam bubble's are nearly spherical. For wet foams, instead of particle-particle interactions, with particle deformation, various stiffness coefficients and friction, we must consider bubble-bubble interactions, with bubble deformation, surface tension and shearing of liquid films (Gardiner *et al.* 2000). The application of homogenisation to foam has enabled our first extension of the homogenisation procedure to 3D, as it removes the complications arising from particle rotation. Moreover, it has provided us with valuable experience and insights for future extensions of the dry granular models to include a finite liquid content. A finite liquid content results in a rate dependency, which is not found in our current dry granular constitutive laws. However, future extension of our granular laws to real soils will necessarily require the introduction of a finite liquid content, along with rate dependent contact laws.

Key Deliverables: **publication [12]**

Project 5. Oscillatory shear of a dry granular material

In this project, the problem of steady and oscillatory shear is being examined as part of the ongoing testing of constitutive models developed in Project 1. Currently, the 1D steady shear flow is being examined, and then later, we will alter the boundary conditions to display an oscillatory behaviour.

The oscillatory shear will be used to examine the irreversibility of microstructural evolution. **Key Deliverables:** a paper in preparation by Tordesillas, A. “Oscillatory shear of a dry granular material” to be submitted to *Powder Technology*.

Project 6. Extension of micropolar homogenisation theory to three-dimensional deformation of polydisperse particulate assemblies

This project is part of the ongoing improvement to the constitutive laws for micropolar granular media and the homogenisation scheme on which they are based. In this project, the homogenisation scheme for micropolar media has been extended to 3D. Furthermore, a first attempt is made at introducing a distribution of particle sizes into our constitutive laws, via the homogenisation scheme. The particle size distribution is introduced into the homogenisation scheme via a contact direction dependent weighting function, which describes the probability of finding a contacting particle of a certain radius in any given direction. Using this new homogenisation scheme, a 3D constitutive equation for dry granular materials, with a particle size distribution, was developed.

Key Deliverables: publication [13]

Project 7 Integrating thermomechanics theory to micromechanical modelling of granular media

Using the principles and techniques of thermomechanics, we set about to derive generalized Cosserat or micropolar constitutive laws in conjunction with the homogenisation method that we developed for granular media[1]. Specific stress-strain relations can be derived from these constitutive laws by setting the form of two functions – the free energy and the dissipation function. This integrated *thermo-micro*-mechanics approach has three main advantages over previous micromechanical techniques developed by our group in the preceding projects 1-6:

- The models developed within this framework are guaranteed to be thermodynamically viable. This eliminates the need for checking that the models are objective and obey the laws of conservation of energy after their development. Furthermore, because no additional constraints need to be introduced, the generalised constitutive relations describe the broadest possible range of thermodynamically valid models.
- Experiments have shown that local fluctuations in both particle displacements and rotations bear a significant influence on the bulk behaviour of granular media – even at low to moderate strains. The models developed here can account for the effects of these local scale fluctuations as well as other important state variables that cannot be readily accounted for in non-thermomechanics based methods.
- Within these constitutive laws there is a clear delineation between the energy dissipated and the energy stored within the granular assembly. This is in contrast with pre-existing models where the stored and dissipated energies are treated as a combined entity.

Key Deliverables: publications [2,3,5,10], conference presentations [18,21]

Project 8 Implementation of thermo-micromechanical constitutive models to finite element simulations of engineering scale problems

In many areas of engineering, Finite Element (FEM) computer simulations are generally considered an essential prerequisite to the implementation of any large investment process. Yet while the application of FEM simulations is gradually growing more popular within particulate industries, simulations have yet to reach the same ubiquitous usage as in, for example, the metal or plastic sheet forming industries. One reason for this technology lag is the difficulty associated with the characterisation and modelling of these materials. This project aims to address this via the implementation of our most advanced model of granular media in finite element simulations of *granular-solid* contact systems including sand-tire interaction.

Key Deliverables: publications [3], conference presentations [21], a paper in preparation by Walsh, SDC and Tordesillas, A. “Finite element implementation of micropolar models of granular

media” to be submitted to *International Journal for Numerical and Analytical Methods in Geomechanics*.

6. Technology Transfer

Our group is collaborating with Dr. John Peters of the Geotechnical Laboratory at ERDC on a number of projects relating to this grant. Technology transfer was achieved through the following research visits, product and activities:

Collaborative Research visits

- (a) Dr. Tordesillas visited ERDC and attended 14th ISTVS conference

Date of visit: October 17-25, 2002

Reason for visit: to participate in the 14th International Conference of the International Society for Terrain-Vehicle Systems which was organized jointly by CRREL and the WES-Mobility Systems Division. Drs Tordesillas and Peters also used this visit to finalize the paper that was submitted to *Geotechnique*.

- (b) Dr Peters attended ICIAM’03, Sydney Australia.

Date of visit: July 7-11, 2003

Reason for visit: Dr John Peters was one of the invited speakers for the series symposia on the “Mathematics and Mechanics of Granular Media” at the 2003 International Congress on Industrial and Applied Mathematics. The series symposia was organized by Dr. Tordesillas.

- (c) Dr Peters visited the Dep. of Mathematics and Statistics, University of Melbourne

Date of visit: July 12-19, 2003

Reason for visit: The objective of this visit is to discuss two major projects that our group is conducting in collaboration with Dr Peters. The first of these is aimed at the development of a graphical environment which will allow researchers to visualise and analyse data produced by, primarily, existing particle simulation programs from ERDC. A team of 16 final year software engineering students from the University of Melbourne is developing this software for Drs Peters and Tordesillas. A prototype of this software was presented to Dr Peters during this visit. The second project is a broad research program encompassing a number of approaches toward developing a micromechanical theory of granular media. During this visit, Dr. Peters was also involved in discussions with another visiting academic, Professor Hide Sakaguchi, from the Earthquake Research Institute of The University of Tokyo.

Numerical code: Completion and hand-over of the **Particle Visualisation Software (PVS)** code to Dr Peters (ERDC) was achieved last November, 2003.

Models: A number of constitutive models of granular media were recently developed and presented in joint publications with Dr Peters.

Research project briefings: Research project briefings took place during the ICIAM’03 conference and at the University of Melbourne. Copies of the powerpoint presentations for some of the oral presentations were provided to Dr Peters.

7. Publications

7 Published papers in peer-reviewed journals

- [1] Gardiner, B and Tordesillas, A. (2004) “Micromechanics of shear bands” *International Journal of Solids and Structures*, in press
- [2] Walsh, SDC and Tordesillas, A (2004) “A thermomechanical approach to the development of micropolar constitutive models for granular media” *Acta Mechanica*, in press

- [3] Tordesillas, A, Walsh, SDC, Gardiner, B (2004) "Bridging the length scales: micromechanics of granular media" *BIT Numerical Mathematics*, in press
- [4] Tordesillas, A, Peters, J., and Gardiner, B (2004) "Shear band evolution and accumulated microstructural development in Cosserat media" *International Journal for Numerical and Analytical Methods in Geomechanics*, in press
- [5] Walsh, S and Tordesillas, A (2004) "The stress response of a semi-infinite granular material subject to a normal point force at the boundary" *Granular Matter*, in press
- [6] Gardiner, B and Tordesillas, A and Peters, JF. (2003) "Incorporating microstructural evolution into micromechanically based continuum models of dry granular media" *Proceedings of the Workshop on the Quasi-Static Deformations of Particulate Materials (QuaDPM'03)* pp. 107-118.
- [7] Tordesillas, A. and Walsh, S. (2002) "Incorporating rolling resistance and contact anisotropy in micromechanical models of granular media", *Powder Technology* **124** (1-2) pp. 106-111.
- [8] Tordesillas, A, Peters, J., and Gardiner, B "Insights on 1D localisation theory and micromechanical constitutive laws" *Geotechnique*

5 Submitted papers in peer-reviewed journals and conference proceedings

- [9] Gardiner, B and Tordesillas, A. "Micromechanical constitutive modelling of granular media: a focus on evolution and loss of contacts in particle clusters." *Journal of Engineering Mathematics*
- [10] Tordesillas, A, Walsh, SDC, Gardiner, B "Micromechanics meets thermomechanics: micropolar models of granular media" *Proceedings, International Congress for Theoretical and Applied Mechanics*
- [11] Muthuswamy, M, Walsh, SDC, Tordesillas, A "An examination of particle rotations in deformations of a granular material – implications for thermo-micromechanical continuum modelling" *Proceedings, International Congress for Theoretical and Applied Mechanics*
- [12] Gardiner, B, Zammit, D and Tordesillas, A. "A micromechanical continuum model for three-dimensional 'granular' liquid foams" submitted to *Journal of Rheology*.
- [13] Gardiner, B and Tordesillas, A. "The effects of particle size distribution based on a three-dimensional homogenisation scheme for micropolar media" submitted to *Powder Technology*.

9 Papers presented in meetings but not published in conference proceedings

- [14] A. Tordesillas
Plenary Speaker: "Bridging the length scales: micromechanics of granular media"
 SciCADE 2003 International Conference on Scientific Computation And Differential Equations
 Trondheim, NORWAY, June 30 - July 4, 2003
<http://www.math.ntnu.no/scicade/>
- [15] A. Tordesillas

Invited Speaker: “Understanding a granular material as a continuum”

Symposia 1/IV “The Mathematics and Mechanics of Granular Media”

ICIAM’03 The 5th International Congress on Industrial and Applied Mathematics

Sydney, AUSTRALIA, July 7-11, 2003

http://www.iciam.org/iciamHome/iciamHome_tf.html

[16] Tordesillas, A.

Invited Speaker: “The anatomy of failure in particulate media”

“Advanced Problems in Mechanics 2003” Institute for Problems in Mechanical Engineering of the Russian Academy of Sciences

St. Petersburg RUSSIA, June 23 - July 2, 2003

[17] Gardiner, B and Tordesillas, A.

“Granular disorder and localisation”

Session on *Granular Systems* , ICIAM’03 The 5th International Congress on Industrial and Applied Mathematics

Sydney, AUSTRALIA, July 7-11, 2003

[18] Walsh, S and Tordesillas, A.

“A thermomechanical approach to the modelling of micropolar media”

Session on *Granular Systems* , ICIAM’03 The 5th International Congress on Industrial and Applied Mathematics

Sydney, AUSTRALIA, July 7-11, 2003

[19] Arber, D and Tordesillas A

“Separating salt and pepper: axial segregation of granular media”

Session on *Education in Mathematics and Computational Science* , ICIAM’03 The 5th International Congress on Industrial and Applied Mathematics

Sydney, AUSTRALIA, July 7-11, 2003

[20] Muthuswamy, M , Arber, D and Tordesillas A

“Adventures with the Shear Band: discrete element simulations of particulate media”

Session on *Multiscale Phenomena* , ICIAM’03 The 5th International Congress on Industrial and Applied Mathematics

Sydney, AUSTRALIA, July 7-11, 2003

[21] Walsh, S and Tordesillas, A.

“The stress-response of a granular material under a rigid flat punch”

ANZIAM 2004, February 2004

8. Awards/Honors Received in 2002

- (1) Tordesillas, A – Appointed member of **Particulate Science and Technology (PST) Network**. This is an Australia wide research network linking 30 world-recognized researchers as members with different expertise from within Australia.
- (2) Tordesillas, A – Appointed committee member, **Mathematics in Science and Society (MASS) Network**. MASS comprises of 30 internationally recognised researchers from mathematics departments in all major Australian Universities.

- (3) Tordesillas, A. – Appointed as Chair of the Organizing Committee for ICIAM'03 The 5th International Congress on Industrial and Applied Mathematics: *Symposia on "The Mathematics and Mechanics of Granular Media"*, Sydney, AUSTRALIA, July 7-11, 2003.
http://www.iciam.org/iciamHome/iciamHome_tf.html
- (4) Tordesillas, A. – Plenary speaker, SciCADE 2003 International Conference on Scientific Computation And Differential Equations, Trondheim, NORWAY, June 30 - July 4, 2003.
<http://www.math.ntnu.no/scicade/>
- (5) Tordesillas, A. and Gardiner, B. Melbourne Research and Development Grant Scheme (MRDGS) award on the project "Statistical mechanics and homogenisation methods for modelling granular media".
- (6) Tordesillas, A. Invited Speaker (but did not attend), "Advanced Problems in Mechanics 2003" <http://www.eng.abdn.ac.uk/~apm/2003> Institute for Problems in Mechanical Engineering of the Russian Academy of Sciences. June 23 - July 2, 2003, St. Petersburg RUSSIA
- (7) Muthuswamy, M. – Received the inaugural AMSI (Australian Mathematical Sciences Institute) student award <http://www.amsi.org.au/index.html> (see her research topic in the attached newsletter "Research Matters")
- (8) Walsh, S – Awarded the 2002 Australian Postgraduate Award and the 2003 Postgraduate Scholarship in Mechanics (PhD Confirmation Feb 28, 2003).
- (9) Arber, D. 2003 Faculty of Science, Dean's Honour List
- (10) Arber, D. 2002 Awarded the 2002 Dept. of Mathematics and Statistics Vacation Scholarship & the 2003 Undergraduate Scholarship in Mechanics
- (11) Muthuswamy, M. 2003 Faculty of Science Dean's Honour List; Faculty of Engineering Dean's Honour List
- (12) Muthuswamy, M. Awarded the 2002 Dept. of Mathematics and Statistics Vacation Scholarship & the 2003 Undergraduate Scholarship in Mechanics
- (13) Tordesillas, A; Walsh, S; Arber, D; and Muthuswamy, M. Invited guest speakers "Going against the grain" The University of Melbourne School Mathematics Competition (Oct 5, 2002) <http://www.mathscomp.ms.unimelb.edu.au/>
- (14) Tordesillas, Walsh, Arber and Muthuswamy Invited guest speakers "See a world in grains of sand" INNOVATION Science Summer School (Dec 3, 2002) <http://www.trinity.unimelb.edu.au/summerschool/science/>

9. Expected Outcomes, New Methodologies and Technologies

Processes involving granular materials are complex and rarely reach more than 60% of the design capacity, due to inadequate understanding of granular rheology. The short term benefits of the proposed project are: improved insights on the rheology of granular media; experimentally validated micromechanical constitutive models with unmatched predictive capabilities; modelling techniques in the analysis of multiscale processes, germane to the Science of Complex Materials. The long term benefits are models of the required reliability for computer-aided design, production and management of particulate and geotechnical systems and processes.

Analytical and numerical simulation tools

- New homogenisation techniques transferable to other complex media and composites
- Experimentally validated 2D constitutive models of granular materials, with particle size and shape distributions, for large strain, dynamic analysis
- Experimentally validated 3D constitutive models for polydisperse assemblies of spherical particles for small strain dynamic analysis
- Finite element (FEM) code for implementing these newly developed constitutive models

Physical insights

- New insights on strength and failure mechanisms in granular materials
- New insights on granular flow and deformation properties around contacting solids (deformable and rigid)
- Improved understanding of microstructural evolution in deforming granular assemblies

Relevance to “Advanced Materials” and “Complex Systems”

A granular material is the ultimate paradigm of a complex system in which simple units (i.e. two contacting particles) behave – collectively – in complicated ways (e.g. segregation and other pattern formation). Modern materials science has exploited this property, with its ever-increasing focus on design, optimisation and synthesis of novel complex systems with tailor made chemical or physical properties. Whether man-made or natural, the bulk behaviour of a given complex material is governed by its microstructure. Therefore, the central challenge in the research and engineering of complex materials is one of quantifying this multiscale spatial and temporal connection [2,3,6,9].

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